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Stochastic Power Control for Time-Varying Long-Term Fading Wireless Networks

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A new time-varying (TV) long-term fading (LTF) channel model which captures both the space and time variations of wireless systems is developed. The proposed TV LTF model is based on a stochastic differential equation driven by Brownian motion. This model is more realistic than the static models usually encountered in the literature. It allows viewing the wireless channel as a dynamical system, thus enabling well-developed tools of adaptive and nonadaptive estimation and identification techniques to be applied to this class of problems. In contrast with the traditional models, the statistics of the proposed model are shown to be TV, but converge in steady state to their static counterparts. Moreover, optimal power control algorithms (PCAs) based on the new model are proposed. A centralized PCA is shown to reduce to a simple linear programming problem if predictable power control strategies (PPCS) are used. In addition, an iterative distributed stochastic PCA is used to solve for the optimization problem using stochastic approximations. The latter solely requires each mobile to know its received signal-to-interference ratio. Generalizations of the power control problem based on convex optimization techniques are provided if PPCS are not assumed. Numerical results show that there are potentially large gains to be achieved by using TV stochastic models, and the distributed stochastic PCA provides better power stability and consumption than the distributed deterministic PCA.

1. INTRODUCTION

Power control (PC) is important to improve the performance of wireless communication systems. The benefits of power minimization are not just increased battery life, but also increased overall network capacity. Users only need to expand sufficient power for acceptable reception, as determined by their quality of service (QoS) specifications, that is usually characterized by the signal-to-interference ratio (SIR) [1]. The majority of research papers in this field use time-invariant (static) models for the wireless channels. In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with time-varying (TV) models, where the channel dynamics become TV stochastic processes [2–6]. TV models take into account the relative motion between transmitters and receivers and temporal variations of the propagating environment such as moving scatterers [1].

Radio channels experience both long-term fading (LTF) and short-term fading (STF). LTF is modeled by lognormal distributions and STF is modeled by Rayleigh or Ricean distributions [7]. In general, LTF and STF are considered superimposed and may be treated separately [7, 8]. In this paper, we consider dynamical modeling and power control for LTF channels that predominate in suburban areas. The STF case has been considered in [2]. In particular, we develop a TV model based on a stochastic differential equation (SDE) driven by Brownian motion for LTF channels. The proposed SDE model is a generalization of the standard lognormal model. In particular, it is shown that the statistics of the SDE model are TV and converge in steady state to their static lognormal counterparts. The proposed model exhibits more realistic behaviors of wireless channels than the current LTF models. It allows viewing the wireless channel as a dynamical system that shows how the channel evolves in time and space. In addition, it allows well-developed tools of adaptive and nonadaptive estimation and identification (to estimate the model parameters) to be applied to this class of problems [9–11]. Finally, based on the proposed TV model, centralized and iterative distributed PCAs are developed.

Power control algorithms (PCAs) can be classified as centralized and distributed. The centralized PCAs require global out-of-cell information available at base stations. The distributed PCAs require base stations to know only in-cell information, which can be easily obtained by local measurements. The power allocation problem has been studied extensively as an eigenvalue problem for nonnegative matrices [12, 13], resulting in iterative PCAs that converge each user's power to the minimum power [14–17], and as optimization-based approaches [18]. Much of these previous works deal with static time-invariant channel models. The scheme introduced in [18], whereby the statistics of the received SIR are used to allocate power, rather than an instantaneous SIR. Therefore, the allocation decisions can be made on a much slower time scale. Previous attempts at capacity determinations in CDMA systems have been based on a "load balancing" view of the PC problem [19]. This reflects an essentially static or at best quasistatic view of the PC problem, which largely ignores the dynamics of channel fading as well as user mobility.

Stochastic PCAs (SPCAs) that use noisy interference estimates have been introduced in [20], where conventional matched filter receivers are used. There, it is shown that the iterative stochastic PCA, which uses stochastic approximations, converges to the optimal power vector under certain assumptions on the stepsize sequence. These results were later extended to the cases where a nonlinear receiver or a decision feedback receiver is used [21]. However, the channel gains are assumed to be fixed ignoring the effects of time-variations on the performance of the system. In this paper, the proposed distributed stochastic PCA is different from those in [20–22] in that these algorithms are based on the assumption that two parameters are assumed to be known at each transmitter, namely, the received matched filter output (received SIR) at its intended receiver and the channel gain between the transmitter and its intended receiver. In the proposed algorithm, only the received SIR at its intended receiver is required.

Other results that attempt to recognize the time-correlated nature of signals are proposed in [23], where blocking is defined via the sojourn time of global interference above a given level. Downlink PC for fading channels is studied in [24] by a heavy traffic limit where averaging methods are used. Stochastic control approach for uplink lognormal fading channels is studied in [25], in which a bounded rate power adjustment model is proposed. Recent work on dynamic PC with stochastic channel variation can be found in [26–28]. However, in our proposed approach, the modeling and analysis of PC strategies investigated here employ wireless models which are TV and subject to fading.

Two different PCAs are proposed. The first one is centralized and based on predictable power control strategies (PPCS) that were first introduced in [2]. PPCS simply mean updating the transmitted powers at discrete times and maintaining them fixed until the next power update begins. The PPCS algorithm is proven to be effectively applicable to such dynamical models for an optimal PC. The outage probability (OP) is used as a performance measure. A distributed version of this algorithm is derived along the lines of [15–17]. The latter helps in allowing autonomous execution at the node or link level, requiring minimal usage of network communication resources for control signaling. The second one is an iterative and distributed SPCA based on stochastic approximations. It requires less information than the SPCAs proposed in [20–22]. Numerical results are provided to evaluate the performance of the proposed PCAs. Since few temporal or even spatiotemporal dynamical models have so far been investigated with the application of any PCA, the suggested dynamical models and PCAs will thus provide a far more realistic and efficient optimum control of wireless channels.

The paper is organized as follows. In Section 2, a TV LTF channel model in which the evolution of the channel is described by an SDE is introduced. In Section 3, several PCAs are discussed. In Section 3.1, a centralized deterministic PCA is proposed in which the solution is obtained through linear programming using PPCS, and then an iterative version is introduced to simplify the implementation of the proposed PCA. A distributed SPCA is proposed in Section 3.2. More general PC cases are presented in Section 3.3. In Section 4, numerical results are presented. Finally, Section 5 provides the conclusion.

2. TIME-VARYING LOGNORMAL FADING CHANNEL MODEL

Wireless communication networks are subject to time-spread (multipath), Doppler spread (time variations), path loss, and interference seriously degrading their performance. In addition to the exponential power path loss, wireless channels suffer from stochastic STF due to multipath, and LTF due to shadowing depending on the geographical area. If a mobile happens to be in some less populated area with few buildings, vehicles, mountains, and so forth, its signal undergoes LTF (lognormal shadowing) [7], which must be compensated in any design. Before introducing the dynamical TV LTF channel model that captures both space and time variations, we first summarize and interpret the traditional lognormal shadowing model, which serves as a basis in the development of the subsequent TV model. The *traditional* (time-invariant) power loss (PL) in dB for a given path is given by [7]

$$\mathrm{PL}(d)[\mathrm{dB}] := \overline{\mathrm{PL}}(d_0)[\mathrm{dB}] + 10\alpha\log\left(\frac{d}{d_0}\right) + \widetilde{Z}, \quad d \geq d_0, \tag{1}$$

where $\overline{\mathrm{PL}}(d_0)$ is the average PL in dB at a reference distance d_0 from the transmitter, the distance d corresponds to the transmitter-receiver separation distance, α is the path loss exponent which depends on the propagating medium, and $\widetilde{Z}$ is a zero-mean Gaussian distributed random variable, which represents the variability of PL due to numerous reflections and possibly any other uncertainty of the propagating environment from one observation instant to the next. The average value of the PL described in (1) is

$$\overline{\mathrm{PL}}(d)[\mathrm{dB}] := \overline{\mathrm{PL}}(d_0)[\mathrm{dB}] + 10\alpha\log\left(\frac{d}{d_0}\right), \quad d \geq d_0. \tag{2}$$

In the traditional models the statistics of the PL do not depend on time t, therefore these models treat PL as static (time invariant). They do not take into consideration the relative motion between the transmitter and the receiver, or variations of the propagating environment due to mobility, appearance, and disappearance of various scatters along the way from one instant to the next. Such spatial and time variations of the propagating environment are captured herein by modeling the PL and the envelop of the received signal as random processes that are functions of space and time. Moreover, and perhaps more importantly, traditional models do not take into consideration the correlation properties of the PL in space and at different observation times. In reality, such correlation properties exist, and one way to model them is through stochastic processes, which obey specific type of stochastic differential equations (SDEs).

In transforming the static model to a dynamical model, the random PL in (1) is relaxed to become a random process, denoted by $\{X(t,\tau)\}_{t\geq 0,\,\tau\geq\tau_0}$, which is a function of both time t and space represented by the time delay τ, where $\tau = d/c$, d is the path length, c is the speed of light, $\tau_0 = d_0/c$, and d_0 is the reference distance. The signal attenuation is defined by $S(t,\tau) \triangleq e^{kX(t,\tau)}$, where $k = -\ln(10)/20$. For simplicity, we first introduce the TV lognormal model for a fixed transmitter-receiver separation distance d (or τ) that captures the temporal variations of the propagating environment. After that we generalize it by allowing both t and τ to vary, as the transmitter and receiver, as well as scatters, are allowed to move at variable speeds. This induces spatiotemporal variations in the propagating environment.

When τ is fixed, the proposed model captures the dependence of $\{X(t,\tau)\}_{t\geq 0,\,\tau\geq\tau_0}$ on time t. This corresponds to examining the time-variations of the propagating environment for fixed transmitter-receiver separation distance. The process $\{X(t,\tau)\}_{t\geq 0,\,\tau\geq\tau_0}$ represents how much power the signal looses at a particular location as a function of time. However, since for a fixed distance d, the PL should be a function of distance, we choose to generate $\{X(t,\tau)\}_{t\geq 0,\,\tau\geq\tau_0}$ by a mean-reverting version of a general linear SDE given by [3]

$$\begin{aligned} dX(t,\tau) &= \beta(t,\tau)(\gamma(t,\tau) - X(t,\tau))\,dt + \delta(t,\tau)\,dW(t), \\ X(t_0,\tau) &= N(\overline{\text{PL}}(d)[\text{dB}];\sigma_{t_0}^2), \end{aligned} \tag{3}$$

where $\{W(t)\}_{t\geq 0}$ is the standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t_0,\tau)$, $N(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{\text{PL}}(d)[\text{dB}]$ is the average path loss in dB. The parameter $\gamma(t,\tau)$ models the average time-varying PL at distance d from the transmitter, which corresponds to $\overline{\text{PL}}(d)[\text{dB}]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)(\gamma(t,\tau) - X(t,\tau))$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift. The initial condition of $X(t,\tau)$ can be obtained from a geometric Brownian motion model which calculates $X(t_0,\tau)$ for a fixed $t = t_0$ as a function of τ.

Let $\{\theta(t,\tau)\}_{t\geq 0} \triangleq \{\beta(t,\tau),\gamma(t,\tau),\delta(t,\tau)\}_{t\geq 0}$. If the random processes in $\{\theta(t,\tau)\}_{t\geq 0}$ are measurable and bounded, then (3) has a unique solution for every $X(t_0,\tau)$ given by [4]

$$\begin{aligned} X(t,\tau) = e^{-\beta([t,t_0],\tau)}\Big(X(t_0,\tau) + \int_{t_0}^{t} e^{\beta([u,t_0],\tau)}\big(\beta(u,\tau)\gamma(u,\tau)\,du \\ + \delta(u,\tau)\,dW(u)\big)\Big), \end{aligned} \tag{4}$$

where $\beta([t,t_0],\tau) \triangleq \int_{t_0}^{t}\beta(u,\tau)\,du$. Moreover, using Ito's stochastic differential rule on $S(t,\tau) = e^{kX(t,\tau)}$ the attenuation coefficient obeys the following SDE:

$$\begin{aligned} dS(t,\tau) = S(t,\tau)\bigg[\Big(k\beta(t,\tau)\Big[\gamma(t,\tau) - \frac{1}{k}\ln S(t,\tau)\Big] \\ + \frac{1}{2}k^2\delta^2(t,\tau)\Big)dt + k\delta(t,\tau)\,dW(t)\bigg], \\ S(t_0,\tau) = e^{kX(t_0,\tau)}. \end{aligned} \tag{5}$$

This model captures the temporal variations of the propagating environment as the random parameters $\{\theta(t,\tau)\}_{t\geq 0}$ can be used to model the TV characteristics of the channel for the particular location τ. A different location is characterized by a different set of parameters $\{\theta(t,\tau)\}$.

Now, let us consider the special case when the parameters $\theta(t,\tau) = \theta(\tau) \triangleq \{\beta(\tau),\gamma(\tau),\delta(\tau)\}$ are time-invariant. In this case we need to show that the expected value of the dynamic PL $X(t,\tau)$, denoted by $E[X(t,\tau)]$, converges to the traditional average PL in (2). In this case, the solution of the SDE (3) is given by

$$\begin{aligned} X(t,\tau) = e^{-\beta(\tau)(t-t_0)}\Big(X(t_0,\tau) + \gamma(\tau)\big(e^{\beta(\tau)(t-t_0)} - 1\big) \\ + \delta(\tau)\int_{t_0}^{t} e^{\beta(\tau)(u-t_0)}\,dW(u)\Big), \end{aligned} \tag{6}$$

where for a given set of time-invariant parameters $\theta(\tau)$ and if the initial $X(t_0,\tau)$ is Gaussian or fixed, the distribution of $X(t,\tau)$ is Gaussian with mean and variance given by

$$\begin{aligned} E[X(t,\tau)] &= e^{-\beta(\tau)(t-t_0)}\Big(E[X(t_0,\tau)] + \gamma(\tau)\big(e^{\beta(\tau)(t-t_0)} - 1\big)\Big), \\ \text{Var}\,[X(t,\tau)] &= \delta(\tau)^2\Big(\frac{1 - e^{-2\beta(\tau)(t-t_0)}}{2\beta(\tau)}\Big) \\ &\quad + e^{-2\beta(\tau)(t-t_0)}\,\text{Var}\,(X(t_0,\tau)). \end{aligned} \tag{7}$$

Expression (7) of the mean and variance shows that the statistics of the communication channel vary as a function of both time t and space τ. As the observation instant t becomes large, the random process $\{X(t,\tau)\}$ converges to a Gaussian random variable with mean $\gamma(\tau) = \overline{\text{PL}}(d)[\text{dB}]$ and variance $\delta(\tau)^2/2\beta(\tau)$. Therefore, the traditional lognormal model (1) is a special case of the general TV LTF model (3). Moreover, the distribution of $S(t,\tau) = e^{kX(t,\tau)}$ is lognormal with mean and variance given by

$$E[S(t,\tau)] = \exp\left(\frac{2kE[X(t,\tau)] + k^2 \operatorname{Var}[X(t,\tau)]}{2}\right),$$
$$\operatorname{Var}[S(t,\tau)] = \exp\left(2kE[X(t,\tau)] + 2k^2 \operatorname{Var}[X(t,\tau)]\right) - \exp\left(2kE[X(t,\tau)] + k^2 \operatorname{Var}[X(t,\tau)]\right). \tag{8}$$

Now, let us go back to the more general case in which $\{\theta(t,\tau)\}_{t\geq 0} \triangleq \{\beta(t,\tau), \gamma(t,\tau), \delta(t,\tau)\}_{t\geq 0}$. At a particular location τ, the mean of the PL process $E[X(t,\tau)]$ is required to track the time variations of the average PL. This can be seen in the following example.

Example 1. Let

$$\gamma(t,\tau) = \gamma_m(\tau)\left(1 + 0.15e^{-2t/T}\sin\left(\frac{10\pi t}{T}\right)\right), \tag{9}$$

where $\gamma_m(\tau)$ is the average PL at a specific location τ, T is the observation interval, $\delta(t,\tau) = 1400$, and $\beta(t,\tau) = 225000$ (these parameters are determined from experimental measurements as will be shown at the end of this section), where for simplicity $\delta(t,\tau)$ and $\beta(t,\tau)$ are chosen to be constant, but in general they are functions of both t and τ. The variations of $X(t,\tau)$ as a function of distance and time are represented in Figure 1. The temporal variations of the environment are captured by a TV $\gamma(t,\tau)$ which fluctuates around different average PLs $\gamma_m' s$, so that each curve corresponds to a different location. It is noticed in Figure 1 that as time progresses, the process $X(t,\tau)$ is pulled towards $\gamma(t,\tau)$. The speed of adjustment towards $\gamma(t,\tau)$ can be controlled by choosing different values of $\beta(t,\tau)$.

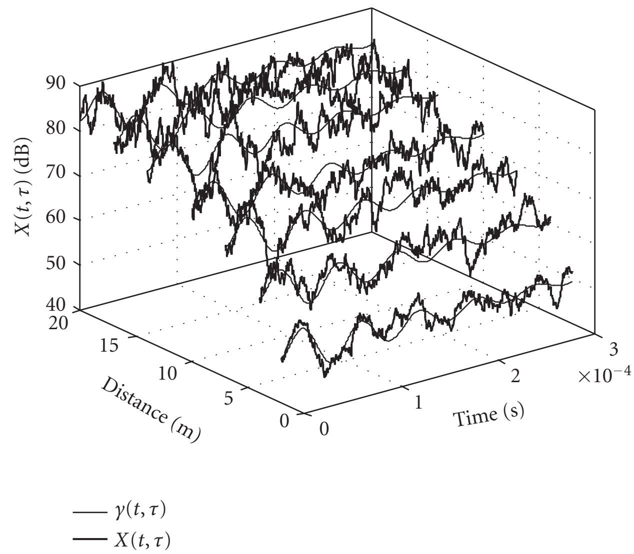

Figure 1: Mean-reverting power path loss as a function of t and τ, for the time-varying $\gamma(t,\tau)$ in Example 1.

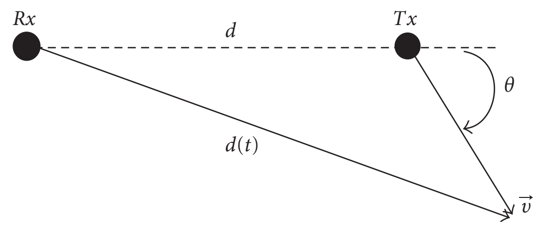

Figure 2: A mobile (transmitter) at a distance d from a base station (receiver) moves with velocity v and in the direction given by θ with respect to the transmitter-receiver axis.

Next, the general spatiotemporal lognormal model is introduced by generalizing the previous model to capture both space and time variations, using the fact that $\gamma(t,\tau)$ is a function of both t and τ. In this case, beside initial distances, the motion of mobiles, that is, their velocities and directions of motion with respect to their base stations, are important factors to evaluate TV PLs for the links involved. This can be illustrated in a simple way for the case of a single transmitter and a single receiver as follows. Consider a base station (receiver) at an initial distance d from a mobile (transmitter) that moves with a certain constant velocity v in a direction defined by an arbitrary constant angle θ, where θ is the angle between the direction of motion of the mobile and the distance vector that starts from the receiver towards the transmitter as shown in Figure 2.

At time t, the distance from the transmitter to the receiver, $d(t)$, is given by

$$d(t) = \sqrt{(d + tv\cos\theta)^2 + (tv\sin\theta)^2} = \sqrt{d^2 + (vt)^2 + 2\,dtv\cos\theta}. \tag{10}$$

Therefore, the average PL at that location is given by

$$\gamma(t,\tau) = \overline{\mathrm{PL}}(d(t))\,[\mathrm{dB}] = \overline{\mathrm{PL}}(d_0)\,[\mathrm{dB}] + 10\alpha\log\frac{d(t)}{d_0} + \xi(t), \quad d(t) \geq d_0, \tag{11}$$

where $\overline{\mathrm{PL}}(d_0)$ is the average PL in dB at a reference distance d_0, $d(t)$ is defined in (10), α is the path loss coefficient, and $\xi(t)$ is an arbitrary function of time representing additional temporal variations in the propagating environment like the appearance and disappearance of additional scatters. The parameter $\gamma(t,\tau)$ is used in the TV lognormal model (3) to obtain a general spatiotemporal lognormal channel model. This is illustrated in the following example.

Example 2. Consider a mobile moving at sinusoidal velocity with average speed 80 km/h, initial distance $d = 50$ meters, $\theta = 135$ degrees, and $\xi(t) = 0$. Figure 3 shows the mean reverting PL $X(t,\tau)$, $\gamma(t,\tau)$, $E[X(t,\tau)]$, velocity of the mobile v, and distance $d(t)$ as a function of time. It can be seen that the mean of $X(t,\tau)$ coincides with the average PL $\gamma(t,\tau)$. Moreover, the variation of $X(t,\tau)$ is due to uncertainties in

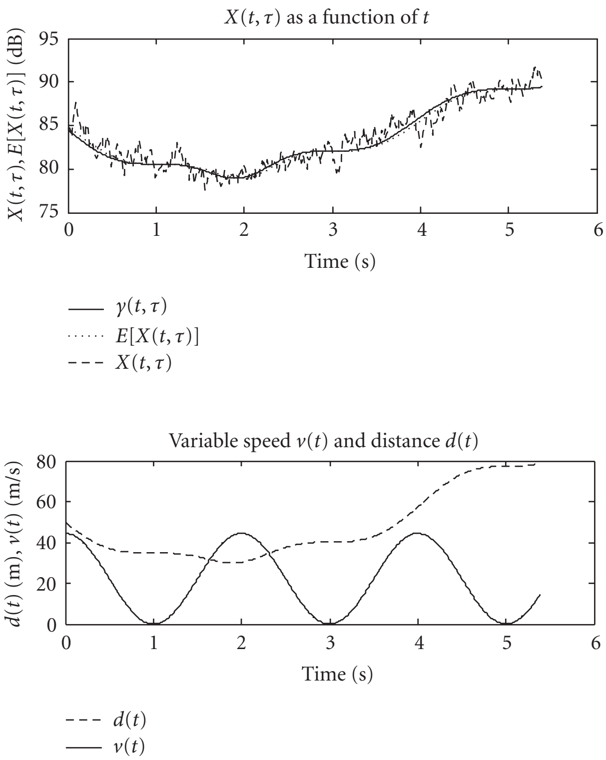

FIGURE 3: Mean-reverting power path loss $X(t, \tau)$ for the TV LTF wireless channel model in Example 2. The mobile starts moving closer to the base station from point 50 meters with an angle of 135 degrees and sinusoidal speed with average 80 km/h (22.2 m/s).

the wireless channel such as movements of objects or obstacles between transmitter and receiver that are captured by the spatiotemporal lognormal models (3) and (11). Additional time variations of the propagating environment, while the mobile is moving, can be captured by using the TV PL coefficient $\alpha(t)$ in (1) in addition to the TV parameters $\beta(t,\tau)$ and $\delta(t,\tau)$, or simply by $\xi(t)$.

Before we finish this section, we want to show that the spatial correlation of the lognormal mean-reverting model of (3) agrees with the experimental spatial correlation [29–31]. In particular, it is reported that the spatial correlation for shadow fading in mobile communications, which compares successfully with experimental data, can be modeled using an exponentially decreasing function multiplied by the variance of the PL process as follows:

$$\mathrm{Cov}_X(\Delta t) \triangleq \sigma_X^2\, e^{-\Delta d/X_c} = \sigma_X^2 e^{-(v/X_c)\Delta t}, \tag{12}$$

where σ_X^2 is the covariance of the PL process, Δd is the distance between two consecutive samples, and v is the velocity of the mobile. X_c is the effective correlation distance which is proportional to the density of the propagating environment corresponding to the distance when the normalized correlation falls to e^{-1} [31]. To show that our spatial dynamical model captures these correlation properties, consider the space-time mean-reverting lognormal model in (3). Without loss of generality, consider the particular case where the parameters $\{\theta(t,\tau)\}_{t\geq 0} = \{\beta(\tau), \gamma(t,\tau), \delta(\tau)\}_{t\geq 0}$. Let $\widetilde{X}(t,\tau) \triangleq X(t,\tau) - E[X(t,\tau)]$, then we have:

$$\begin{gathered} d\widetilde{X}(t,\tau) = -\beta(\tau)\widetilde{X}(t,\tau)dt + \delta(\tau)dW(t), \\ \widetilde{X}(t_0,\tau) = N(0;\sigma_{t_0}^2). \end{gathered} \tag{13}$$

The solution of (13) is given by

$$\widetilde{X}(t,\tau) = e^{-\beta(\tau)(t-t_0)}\left(\widetilde{X}(t_0,\tau) + \int_{t_0}^{t} e^{\beta(\tau)(u-t_0)}\delta(\tau)dW(u)\right). \tag{14}$$

The mean of the process $\widetilde{X}(t,\tau)$ is zero, and its covariance is given by

$$\mathrm{Cov}_{\widetilde{X}}(t,v) = e^{-\beta(\tau)(t+v)}e^{2\beta(\tau)t_0}\left[\sigma_{t_0}^2 + \frac{\delta^2(\tau)}{2\beta(\tau)}\left(e^{2\beta(\tau)(t\wedge v - t_0)} - 1\right)\right], \tag{15}$$

where $t \wedge v \triangleq \min(t, v)$. Letting $v = t + \Delta t$, then

$$\begin{aligned} &\mathrm{Cov}_{\widetilde{X}}(t, t+\Delta t) \\ &\quad = e^{-2\beta(\tau)(t-t_0)-\beta(\tau)\Delta t}\left[\sigma_{t_0}^2 + \frac{\delta^2(\tau)}{2\beta(\tau)}\left(e^{2\beta(\tau)(t-t_0)} - 1\right)\right]. \end{aligned} \tag{16}$$

The covariance of the overall dynamical model indicates what proportion of the environment remains constant from one observation instant or location to the next, separated by the sampling interval. Since the mobile is in motion, it implies that this corresponds to a spatial covariance. If we choose the variance of the initial condition such that $\sigma_{t_0}^2 = \delta^2(\tau)/2\beta(\tau)$, then

$$\mathrm{Cov}_{\widetilde{X}}(t, t+\Delta t) = \frac{\delta^2(\tau)}{2\beta(\tau)}e^{-\beta(\tau)\Delta t} = \sigma_{t_0}^2 e^{-\beta(\tau)\Delta t} \triangleq \mathrm{Cov}_{\widetilde{X}}(\Delta t). \tag{17}$$

Expression (17) indicates that the spatial covariance of our overall dynamical model corresponds to the reported experimental spatial covariance given by (12). The comparison further indicates that $\beta(\tau)$ is a characteristic of both the propagating environment and the separation distance of two consecutive samples, that is, $\beta(\tau)$ is inversely proportional to the density of the propagating environment, and directly proportional to the sample separation distance. Note that the spatial covariance is an important characteristic for our dynamical mean-reverting shadow fading model since it can be clearly used in order to identify the random parameters $\{\beta(\tau), \delta(\tau)\}$. This could be accomplished by using experimental data of $\mathrm{Cov}_{\widetilde{X}}(\Delta t)$. Therefore, the parameters $\{\beta(\tau), \delta(\tau)\}$ can be estimated on-line from experimental measurements. Finally, we note that the variance of the initial condition of the PL process $\sigma_{t_0}^2$ should inevitably increase with distance, or equivalently $\delta(\tau)$ should increase and/or $\beta(\tau)$ decrease.

Subsequently, we consider the uplink channel of a cellular network. We assume that users are already assigned to their base stations and therefore we do not consider the base station assignment case. Let M be the number of mobiles (users), and N the number of base stations. The received signal of the ith mobile at its assigned base station at time t can be expressed as

$$y_i(t) = \sum_{j=1}^{M} \sqrt{p_j(t)} s_j(t) S_{ij}(t) + n_i(t), \tag{18}$$

where $p_j(t)$ is the transmitted power of mobile j at time t, which acts as a scaling on the information signal $s_j(t)$, $n_i(t)$ is the channel disturbance or noise at the base station of mobile i, and $S_{ij}(t)$ is the signal attenuation coefficient between mobile j and the base station assigned to mobile i. Therefore, in a cellular network the spatiotemporal model described in (3) for M mobiles and N base stations can be described as

$$\begin{aligned} dX_{ij}(t,\tau) &= \beta_{ij}(t,\tau)\big(\gamma_{ij}(t,\tau) - X_{ij}(t,\tau)\big)dt + \delta_{ij}(t,\tau)dW_{ij}(t), \\ X_{ij}(t_0,\tau) &= N\big(\overline{\mathrm{PL}}(d)[\mathrm{dB}]_{ij}; \sigma_{t_0}^2\big), \quad 1 \le i,\ j \le M, \end{aligned} \tag{19}$$

and the signal attenuation coefficients $S_{ij}(t)$ are generated using the relation $S_{ij}(t,\tau) = e^{kX_{ij}(t,\tau)}$, where $k = -\ln(10)/20$. Moreover, correlation between the channels in a multiuser/multiantenna model can be induced by letting the different Brownian motions W_{ij}'s to be correlated, that is, $E[\mathbf{W}(t)\mathbf{W}(t)^T] = \mathbf{Q}(\tau)\cdot t$, where $\mathbf{W}(t) \triangleq (W_{ij}(t))$, and $\mathbf{Q}(\tau)$ is some (not necessarily diagonal) matrix that is a function of τ and dies out as τ becomes large.

The TV LTF channel models in (19) are used to generate the link gains for the proposed PCAs introduced in the next section.

3. POWER CONTROL ALGORITHMS

In this section, different PCAs are introduced based on the TV lognormal channel model derived in the previous section. A deterministic PCA (DPCA) is introduced first, and then a stochastic PCA (SPCA) is presented. Both centralized and distributed PCAs are considered.

3.1. *Deterministic power control schemes*

The aim of the PCAs described here is to minimize the total transmitted power of all users while maintaining acceptable quality of service (QoS) for each user. The measure of QoS can be defined by the signal-to-interference ratio (SIR) for each link to be larger than a target SIR. Consider a cellular network as described above, then the centralized PC problem for *time-invariant* channels can be stated as [2]

$$\min_{(p_1\ge 0,\dots,p_M\ge 0)} \sum_{i=1}^{M} p_i \quad \text{subject to } \frac{p_i g_{ii}}{\sum_{j\ne i}^{M} p_j g_{ij} + \eta_i} \ge \varepsilon_i, \quad 1 \le i \le M, \tag{20}$$

where p_i is the power of mobile $i, g_{ij} > 0$ is the time-invariant channel gain between mobile j and the base station assigned to mobile $i, \varepsilon_i > 0$ is the target SIR of mobile i, and $\eta_i > 0$ is the noise power level at the base station of mobile i. The constraint in (20) for the TV lognormal channel models described using path-wise QoS of each user over a time interval $[0,T]$ is given by

$$\frac{\int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt}{\sum_{j\ne i}^{M}\int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt + \int_0^T n_i^2(t)dt} \ge \varepsilon_i, \quad i = 1,\dots,M. \tag{21}$$

Consequently, a natural generalization of the PC problem in (20) with respect to the TV lognormal models in (19) can be written as

$$\begin{aligned} &\min_{(p_1\ge 0,\dots,p_M\ge 0)} \left\{\sum_{i=1}^{M}\int_0^T p_i(t)dt\right\}, \quad \text{subject to} \\ &\sum_{j\ne i}^{M}\int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i}\int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt \\ &\quad + \int_0^T n_i^2(t)dt \le 0, \quad i = 1,\dots,M. \end{aligned} \tag{22}$$

A solution to (22) is presented by first introducing the communication meaning of predictable power control strategies (PPCS). In wireless cellular networks, it is practical to observe and estimate channels at base stations and then send the information back to the mobiles to adjust their power signals $\{p_i(t)\}_{i=1}^{M}$. Since channels experience delays, and power control is not feasible continuously in time but only at discrete time instants, the concept of predictable strategies is introduced [2]. Consider a set of discrete time strategies $\{p_i(t_k)\}_{i=1}^{M}$, where $0 = t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots \le T$. At time t_{k-1}, the base stations observe or estimate the channel information $\{S_{ij}(t_{k-1}), s_i(t_{k-1})\}_{i,j=1}^{M}$. Using the concept of predictable strategy, the base stations determine the control strategy $\{p_i(t_k)\}_{i=1}^{M}$ for the next time instant t_k. The latter is communicated back to the mobiles, which hold these values during the time interval $[t_{k-1}, t_k)$. At time t_k, a new set of channel information $\{S_{ij}(t_k), s_i(t_k)\}_{i,j=1}^{M}$ is observed at the base stations and the time t_{k+1} control strategies $\{p_i(t_{k+1})\}_{i=1}^{M}$ are computed and communicated back to the mobiles which hold them constant during the time interval $[t_k, t_{k+1})$. Such decision strategies are called predictable. More specifically, we say that a discrete time signal $\{\varphi(k);\ k = 0,1,\dots\}$ is predictable with respect to a filtration $\{Z_k\}$ if $\varphi(k)$ is Z_{k-1} measurable. Using the concept of PPCS over any time interval $[t_k t_{k+1}]$, (22) is equivalent to

$$\begin{aligned} &\min_{\mathbf{p}(t_{k+1})>0} \sum_{i=1}^{M} p_i(t_{k+1}), \quad \text{subject to } \mathbf{p}(t_{k+1}) \\ &\quad \ge \mathbf{\Gamma}\mathbf{G}_I^{-1}(t_k,t_{k+1}) \times \big(\mathbf{G}(t_k,t_{k+1})\mathbf{p}(t_{k+1}) + \boldsymbol{\eta}(t_{k+1})\big), \end{aligned} \tag{23}$$

where

$$
\begin{aligned}
g_{ij}(t_k, t_{k+1}) &:= \int_{t_k}^{t_{k+1}} s_j^2(t) S_{ij}^2(t)\,dt,\\
\eta_i(t_k, t_{k+1}) &:= \int_{t_k}^{t_{k+1}} n_i^2(t)\,dt, \quad 1 \le i,\ j \le M,\\
\mathbf{G}_I(t_k, t_{k+1}) &= \operatorname{diag}\left(g_{11}(t_k, t_{k+1}), \dots, g_{MM}(t_k, t_{k+1})\right),\\
\mathbf{G}(t_k, t_{k+1}) &= \begin{cases} 0 & \text{if } i = j,\\ g_{ij}(t_k, t_{k+1}) & \text{if } i \neq j, \end{cases}\\
\boldsymbol{\eta}(t_k, t_{k+1}) &= \left(\eta_1(t_k, t_{k+1}), \dots, \eta_M(t_k, t_{k+1})\right)^{\mathrm{tr}},\\
\mathbf{p}(t_{k+1}) &= \left(p_1(t_{k+1}), \dots, p_M(t_{k+1})\right)^{\mathrm{tr}},\\
\boldsymbol{\Gamma} &= \operatorname{diag}(\varepsilon_1, \dots, \varepsilon_M),
\end{aligned}
\tag{24}
$$

$\operatorname{diag}(\cdot)$ denotes a diagonal matrix with its argument as diagonal entries, and "tr" stands for matrix or vector transpose. The optimization in (23) is a linear programming problem in $M \times 1$ vector of unknowns $\mathbf{p}(t_{k+1})$. Here $[t_k, t_{k+1}]$ is a time interval such that the channel model does not change significantly, that is, $[t_k, t_{k+1}]$ should be smaller than the coherence time of the channel.

Next, we consider an iterative distributed version of the centralized PCA in (23). This is convenient for on-line implementation since it helps autonomous execution at the node or link level, requiring minimal usage of network communication resources for control signaling. The iterative distributed PCA proposed in [15–17] can be used to find a distributed version to the centralized PCA in (23). The constraint in (23) can be rewritten as

$$
\begin{aligned}
&\left(\mathbf{I} - \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k, t_{k+1})\mathbf{G}(t_k, t_{k+1})\right)\mathbf{p}(t_{k+1})\\
&\qquad \ge \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k, t_{k+1})\boldsymbol{\eta}(t_{k+1}).
\end{aligned}
\tag{25}
$$

Defining $\mathbf{F}(t_k, t_{k+1}) \triangleq \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k, t_{k+1})\mathbf{G}(t_k, t_{k+1})$ and $\mathbf{u}(t_k, t_{k+1}) \triangleq \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k, t_{k+1})\boldsymbol{\eta}(t_{k+1})$, then (25) can be rewritten as

$$
\left(\mathbf{I} - \mathbf{F}(t_k, t_{k+1})\right)\mathbf{p}(t_{k+1}) \ge \mathbf{u}(t_k, t_{k+1}). \tag{26}
$$

If channel gains are time invariant, that is, $\mathbf{F}(t_k, t_{k+1}) = \mathbf{F}$ and $\mathbf{u}(t_k, t_{k+1}) = \mathbf{u}$, then the power control problem is feasible if $\rho_{\mathbf{F}} < 1$, where $\rho_{\mathbf{F}}$ is the Perron-Frobenius eigenvalue of $\mathbf{F}$ [15]. It is shown in [15–17] that the following iterative PCA converges to the minimal power vector when $\rho_{\mathbf{F}} < 1$:

$$
\mathbf{p}(t_{k+1}) = \mathbf{F}\mathbf{p}(t_k) + \mathbf{u}. \tag{27}
$$

However, our channel gains are time varying, thus a "time-varying version" of the deterministic PCA (DPCA) in (27) can be defined as

$$
\mathbf{p}(t_{k+1}) = \mathbf{F}(t_k, t_{k+1})\mathbf{p}(t_k) + \mathbf{u}(t_k, t_{k+1}). \tag{28}
$$

Clearly, in general the power vector $\mathbf{p}(t_k)$ will not converge to some deterministic constant as it does in (27). Rather, in a time-varying (random) propagation environment, it is required that the power vector $\mathbf{p}(t_k)$ converges in distribution to a well-defined random variable. Since $\mathbf{F}(t_k, t_{k+1})$ is a random matrix-valued process, the key convergence condition is the Lyapunov exponent $\lambda_{\mathbf{F}} < 0$ [32], where $\lambda_{\mathbf{F}}$ is defined as

$$
\lambda_{\mathbf{F}} = \lim_{k \to \infty} \frac{1}{k} \log \left\| \mathbf{F}(t_0, t_1)\mathbf{F}(t_1, t_2) \cdots \mathbf{F}(t_k, t_{k+1}) \right\|. \tag{29}
$$

Throughout this section, we assume that the PC problem is feasible, that is, there exists a power vector $\mathbf{p}(t_k)$ that satisfies the inequality in (23) for all t_k. The distributed version of (28) can be written as

$$
p_i(t_{k+1}) = \frac{\varepsilon_i(t_k)}{R_i(t_k)} p_i(t_k), \quad i = 1, \dots, M, \tag{30}
$$

where $R_i(t_k)$ is instantaneous SIR defined by

$$
R_i(t_k) = \frac{p_i(t_k) g_{ii}(t_k, t_{k+1})}{\sum_{j \neq i}^{M} p_j(t_k) g_{ij}(t_k, t_{k+1}) + \eta_i(t_k, t_{k+1})}, \quad i = 1, \dots, M. \tag{31}
$$

It is shown in [22] that the performance of the DPCA in (30) in terms of power consumption is not optimal when the channel environment is time varying (random). Actually, the performance can be severely degraded when PCAs that are designed for deterministic channels are applied to TV channels [22]. Therefore, stochastic PCAs (SPCAs) must be used in order to ensure stable optimal power consumption. The latter is introduced in the following section.

3.2. *Stochastic power control schemes*

A distributed SPCA similar to the one described in [20] is used in this section, where the transmit powers are updated based on stochastic approximations. Let us define the instantaneous interference at time t_k by

$$
I_i(t_k) = \sum_{j \neq i}^{M} p_j(t_k) g_{ij}(t_k, t_{k+1}) + \eta_i(t_k, t_{k+1}), \quad i = 1, \dots, M, \tag{32}
$$

then the SPCA proposed in [20], which uses the concept of interference averaging as introduced in [33], can be used to update the transmitted power recursively as

$$
\begin{aligned}
p_i(t_{k+1}) &= \left(1 - a(t_k)\right) p_i(t_k)\\
&\quad + a(t_k) \frac{\varepsilon_i(t_k)}{g_{ii}(t_k, t_{k+1})} \left[I_i(t_k)\right], \quad i = 1, \dots, M,
\end{aligned}
\tag{33}
$$

where $a(t_k)$ is the stepsize at time t_k, which satisfies certain conditions as explained later. Substituting (32) into (33) and using (31) yield

$$p_i(t_{k+1}) = (1 - a(t_k))\,p_i(t_k) + a(t_k)\frac{\varepsilon_i(t_k)}{R_i(t_k)}\,p_i(t_k), \quad i = 1,\dots,M. \tag{34}$$

If the PC problem in (22) is feasible, the distributed SPCA in (34) converges to the optimal power vector when the stepsize sequence satisfies certain conditions. Two different types of convergence results are shown in [34] under different choices of the stepsize sequence. If the stepsize sequence satisfies $\sum_{k=0}^{\infty} a(t_k) = \infty$ and $\sum_{k=0}^{\infty} a(t_k)^2 < \infty$, then the SPCA in (34) converges to the optimal power vector with probability one. However, due to the requirement for the SPCA to track TV environments, the iteration stepsize sequence is not allowed to decrease to zero. So we consider the case where the condition $\sum_{k=0}^{\infty} a(t_k)^2 < \infty$ is violated. This includes the situation when the stepsize sequence decreases slowly to zero, and the situation when the stepsize is fixed at a small constant. In the first case when $a(t_k) \to 0$ slowly, the SPCA in (34) converges to the optimal power vector in probability. While in the second case the power vector clusters around the optimal power. In fact, the error between the power vector and the optimal value does not vanish for nonvanishing stepsize sequence; this is the price paid in order to make the algorithm in (34) able to track TV environments. This algorithm is fully distributed in the sense that each user iteratively updates its power level by estimating the received SIR of its own channel. It does not require any knowledge of the link gains and state information of other users. The remaining three parameters of (34): the user power value in the previous iteration $p_i(t_k)$, its SIR target value $\varepsilon_i(t_k)$, and stepsize sequence $a(t_k)$, are trivially known by the user. It is worth mentioning that the proposed distributed SPCA in (34) is different from the algorithm proposed in [22] where two parameters, namely, the received SIRs $R_i(t_k)$ and the channel gains $g_{ii}(t_k, t_{k+1})$, are required to be known. In contrast, here only the received SIRs $R_i(t_k)$ are required in (34).

The received SIRs $R_i(t_k)$ can be estimated at the base stations every L bits, and then transmitted back to the users. Each user keeps its transmitted power level fixed until the feedback from its base station arrives and then updates its transmitted power according to (34). This process occurs during the time interval $[t_k, t_{k+1}]$ which should be chosen such that the channel model does not change significantly, that is, $[t_k, t_{k+1}]$ should be smaller than the coherence time of the channel. For small $[t_k, t_{k+1}]$, the power control updates will be more frequent and thus convergence will be faster. However, frequent transmission of the feedback on the downlink channel will effectively decrease the capacity of the system since more system resources (bandwidth) will have to be used for power control.

3.3. More generalizations

Without predictable power control strategies, two formulations in terms of convex optimization using linear programming techniques and stochastic control with integral or exponential-of-integral constraints are introduced in this section. Moreover, an alternative stochastic power control formulation that meets outage constraints is also discussed.

The first problem is formulated in terms of convex optimization and linear programming as follows:

$$\min_{(p_1\ge 0,\dots,p_M\ge 0)} \left\{\sum_{i=1}^{M}\int_{t_k}^{t_{k+1}} p_i(t)\,dt\right\}, \quad \text{subject to}$$

$$\sum_{j\neq i}^{M}\int_{t_k}^{t_{k+1}} p_j(t)s_j^2(t)S_{ij}^2(t)\,dt - \frac{1}{\varepsilon_i}\int_{t_k}^{t_{k+1}} p_i(t)s_i^2(t)S_{ii}^2(t)\,dt + \int_{t_k}^{t_{k+1}} n_i^2(t)\,dt \le 0, \quad i = 1,\dots,M. \tag{35}$$

According to the above formulation using predictable strategies, this is a convex optimization problem. In addition, any interval $[0, T]$ can be considered as $0 = t_0 < t_1 < t_2 \cdots < t_k < t_{k+1} < \cdots \le T$, and by approximating the integrals by Riemann sums as close as desired, it can be shown that (35) reduces to a linear programming problem again.

The second problem is formulated in terms of stochastic control with integral or exponential-of-integral constraints as

$$\min_{(p_1\ge 0,\dots,p_M\ge 0)} \left\{\sum_{i=1}^{M} E\int_{0}^{T} p_i(t)\,dt\right\}, \quad \text{subject to}$$

$$J_{0,T}^{i}(p) \triangleq E\left\{\sum_{j\neq i}^{M}\int_{0}^{T} p_j(t)s_j^2(t)S_{ij}^2(t)\,dt - \frac{1}{\varepsilon_i}\int_{0}^{T} p_i(t)s_i^2(t)S_{ii}^2(t)\,dt + \int_{0}^{T} n_i^2(t)\,dt\right\} \le 0, \quad i = 1,\dots,M. \tag{36}$$

If there exists a set of $\{\varepsilon_i\}_{i=1}^{M}$ such that the QoS are feasible, by employing Lagrange multipliers λ_i for each $J_{0,T}^{i}(p)$ we can introduce

$$L^{\lambda}(u^*, \lambda) = \min_{(p_1\ge 0,\dots,p_M\ge 0)} \times \left\{\sum_{i=1}^{M} E\left[\int_{0}^{T} p_i(t)\,dt + \lambda_i\left[\sum_{j\neq i}^{M}\int_{0}^{T} p_j(t)s_j^2(t)S_{ij}^2(t)\,dt - \frac{1}{\varepsilon_i}\int_{0}^{T} p_i(t)s_i^2(t)S_{ii}^2(t)\,dt + \int_{0}^{T} n_i^2(t)\,dt\right]\right]\right\} \tag{37}$$

and then solve the problem $l\ (\lambda^*, u^*) = \sup_{\lambda \geq 0} L^\lambda\ (u^*, \lambda)$. Further, it can be shown that $L^\lambda\ (u^*, \lambda)$ satisfies a dynamic programming equation of the Hamilton-Jacobi-Bellman type [35].

Similarly, the QoS can be considered as point-wise constraints and pursue the problem

$$\min_{(p_1 \geq 0, \ldots, p_M \geq 0)} \left\{ \sum_{i=1}^{M} E \int_0^T p_i(t)dt \right\}, \quad \text{subject to}$$
$$\sum_{j \neq i}^{M} p_j(t)s_j^2(t)S_{ij}^2(t) - \frac{1}{\varepsilon_i} p_i(t)s_i^2(t)S_{ii}^2(t) + n_i^2(t) \leq 0, \qquad t \in [0, T],\ i = 1, \ldots, M. \tag{38}$$

Optimizations (36) and (38) are convex optimization problems, since their objective functions and constraints are convex.

An alternative stochastic power control formulation can be stated in terms of outage probability (OP). It is defined as the probability that a randomly chosen link fails due to excessive interference [12]. Therefore, smaller OP implies larger capacity of the wireless network. A link with a received SIR R_i, less than or equal to a target SIR ε_i, is considered a communication failure. The OP $O(\varepsilon_i)$ is expressed as $O(\varepsilon_i) = \text{Prob}\{R_i \leq \varepsilon_i\}$. The stochastic PC problem that meets outage constraints can be formulated as

$$\min_{(p_1 \geq 0, \ldots, p_M \geq 0)} \left\{ \sum_{i=1}^{M} \int_0^T p_i(t)dt \right\}, \quad \text{subject to}$$
$$\Pr\left\{ \left(\sum_{j \neq i}^{M} \int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i} \int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt + \int_0^T n_i^2(t)dt \right) \geq 0 \right\} \leq O_i, \tag{39}$$

where $t \in [0, T]$, O_i is the target OP of user i, and $i = 1, \ldots, M$. The probabilities in the constraint of (39) are very difficult to compute. Therefore, Chernoff bounds [36] can be used to evaluate the probability of failure to achieve a desired QoS requirement as follows:

$$\begin{aligned} &\Pr\left\{ \left(\sum_{j \neq i}^{M} \int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i} \int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt + \int_0^T n_i^2(t)dt \right) \geq 0 \right\} \\ &\leq E\left\{ \exp\left(c_i \left(\sum_{j \neq i}^{M} \int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i} \int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt + \int_0^T n_i^2(t)dt \right) \right) \right\}, \end{aligned} \tag{40}$$

where $c_i > 0$, $i = 1, \ldots, M$. The Chernoff bound associated with (40) subject to (18) and (19) can be computed in [2] using a version of the backward Kolmogorov equation; the right-hand side of (40) is given by [2]

$$\begin{aligned} &E\left\{ \exp\left(c_i \left(\sum_{j \neq i}^{M} \int_0^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i} \int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt + \int_0^T n_i^2(t)dt \right) \right) \right\} \\ &= \exp\left[\frac{c_i^2}{2} \sigma^2 T \right] V^i(0, x), \end{aligned} \tag{41}$$

where σ^2 is the variance of the noise $n_i(t)$, and $V^i(t, x)$ is defined by

$$\begin{aligned} V^i(t, x) \triangleq E\Bigg\{ \exp\Bigg(c_i \Bigg(&\sum_{j \neq i}^{M} \int_t^T p_j(t)s_j^2(t)S_{ij}^2(t)dt - \frac{1}{\varepsilon_i} \int_t^T p_i(t)s_i^2(t)S_{ii}^2(t)dt + \int_t^T n_i^2(t)dt \Bigg) \\ &- c_i \int_t^T n_i^2(t)dt \Bigg) \mid X_{ij}(0) \Bigg\}. \end{aligned} \tag{42}$$

Thus, the Chernoff bound is computed explicitly in (41), and then has to be minimized over $c_i \geq 0$.

To illustrate the efficiency of the various PCAs proposed in this paper, numerical results are presented in the next section.

4. NUMERICAL RESULTS

In this section, we provide two numerical examples to determine the performance of the various PCAs under the proposed TV LTF channel models. In Example 1, we compare the performance of the centralized DPCA using PPCS described in (23) under two different types of TV LTF channel models; the stochastic TV models in (3) and the static TV models in (1). In the second example, the performance of the distributed DPCA (30) and the distributed SPCA (34) under the proposed stochastic TV LTF channel models is determined.

The cellular model has the following features: the number of transmitters (mobiles) is $M = 24$, the information signal $s_i(t) = 1$ for $i = 1, \ldots, M$, the number of bits L in each power update period is one, initial distances of all mobiles with respect to their own base stations d_{ii} are generated as uniformly independent identically distributed (*i.i.d.*) random variables (*r.v.*'s) in (10–100) meters, cross initial distances of all mobiles with respect to other base stations d_{ij}, $i \neq j$, are generated as uniformly *i.i.d. r.v.*'s in (250–550) meters, the angle θ_{ij} between the direction of motion of mobile j and the distance vector passes through base station i and mobile j are generated as uniformly *i.i.d. r.v.*'s in (0–180) degrees, the average velocities of mobiles are generated as uniformly *i.i.d. r.v.*'s in (40–100) km/h, all mobiles move at

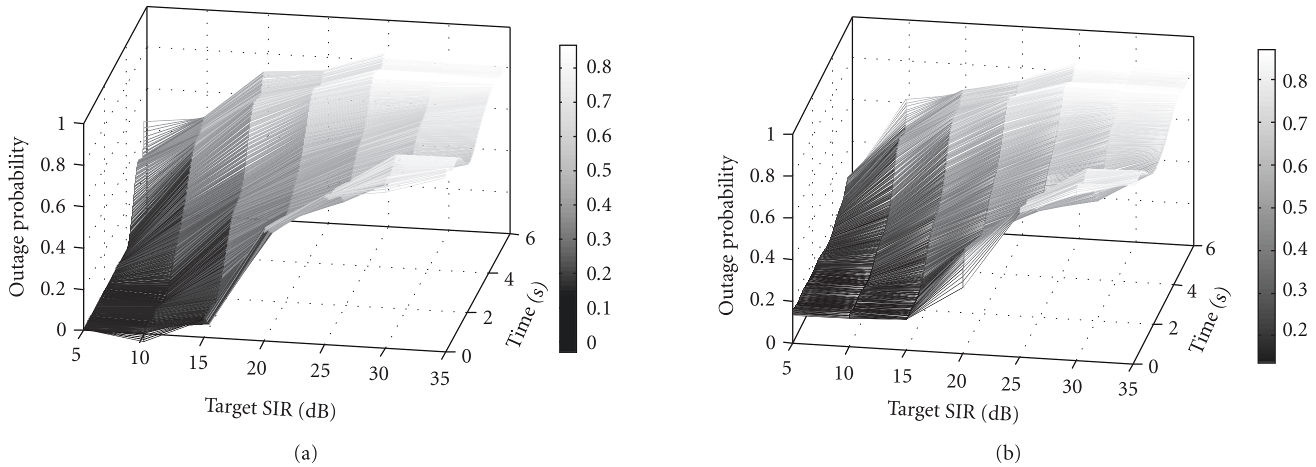

FIGURE 4: OP for the centralized DPCA using PPCS under TV LTF models for (a) stochastic models, (b) static models.

sinusoidal variable velocities around their average velocities such that the peak velocity is two times the average speed, power path loss exponent is 3.5, initial reference distance from each of the transmitters is 10 meters, power path loss at the initial reference distance is 67 dB, $\delta_{ij}(t,\tau) = 1400$ and $\beta_{ij}(t,\tau) = 225000$ for the SDEs, and η_i's are *i.i.d.* Gaussian *r.v.*'s with zero mean and variance = 10^{-12} W. The performance measure is outage probability (OP).

Example 3. In this example, the centralized DPCA using PPCS in (23) is performed on two different TV LTF channel models; the stochastic TV model in (3) and the static model encountered in the literature [12]. It is assumed that the targets SIR ε_i for all users are the same, and varied from 5 dB to 35 dB with step 5 dB. For each value of ε_i the OP is computed every 15 millisecond, that is, $[t_k, t_{k+1}]$ =15 milliseconds. The simulation is performed for 5 seconds. The OP is computed using Monte-Carlo simulations. The OPs for the centralized DPCA using PPCS based on both stochastic and static TV LTF channel models are shown in Figure 4(a) and Figure 4(b), respectively. Figure 4 shows how the OP changes with respect to the target SIR, ε_i, and time. As the target SIR increases the OP increases. This is obvious since we expect more users to fail as ε_i increases. The OP also changes as a function of time, since mobiles move in different directions and velocities. The *average* OP versus ε_i over the whole simulation time (5 seconds) is shown in Figure 5, which shows that the performance of PPCS using the stochastic models is on average much better than static models. For example, at 10 dB target SIR, the OP is reduced from 0.26 for static models to 0.18 for TV stochastic ones; this represents an improvement of over 30%. The PPCS algorithm for stochastic models outperforms the static ones by an order of magnitude. It can be seen that as target SIR, ε_i increases the performance gap between the PPCS using stochastic and static models decreases. This is because the effect of ε_i (required QoS) is dominant.

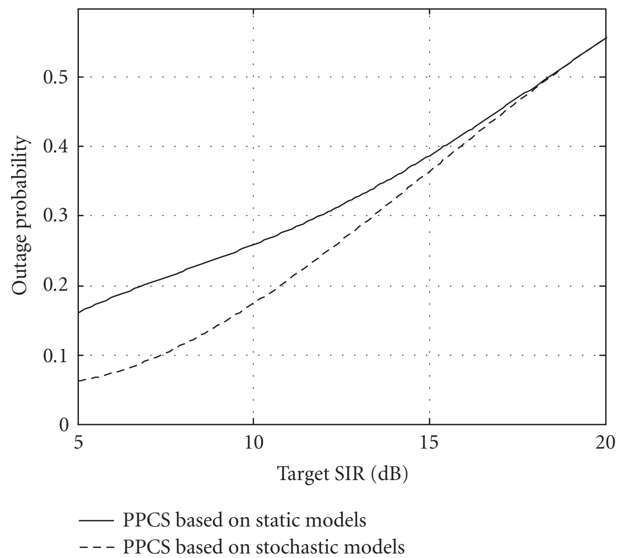

FIGURE 5: Average OP for TV LTF channel models with $\delta(t) = 1400$. Performance comparison.

Figure 6 shows the average OP over the whole simulation time (5 seconds) for higher noise variance ($\delta(t,\tau)$= 2800). In this case the stochastic PL $X(t,\tau)$ have higher variations or fluctuations around the average PL $\gamma(t,\tau)$, since this parameter controls the instantaneous variance of the stochastic PL. The PPCS based on static models when the actual channels have high variance gives higher OP than when the actual channels have low variance as observed in Figures 5 and 6. This is due to the fact that channels with high variance deviate significantly from the average (static) channels. For example, at 10 dB target SIR, the OP in the static case is about 0.32, while in the stochastic case, it is about 0.2, an improvement of over 37%.

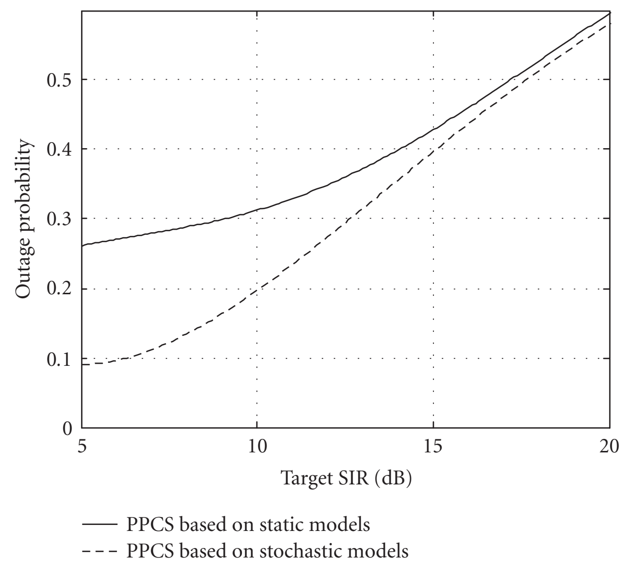

Figure 6: Average OP for TV LTF channel models with $\delta(t) = 2800$. Performance comparison.

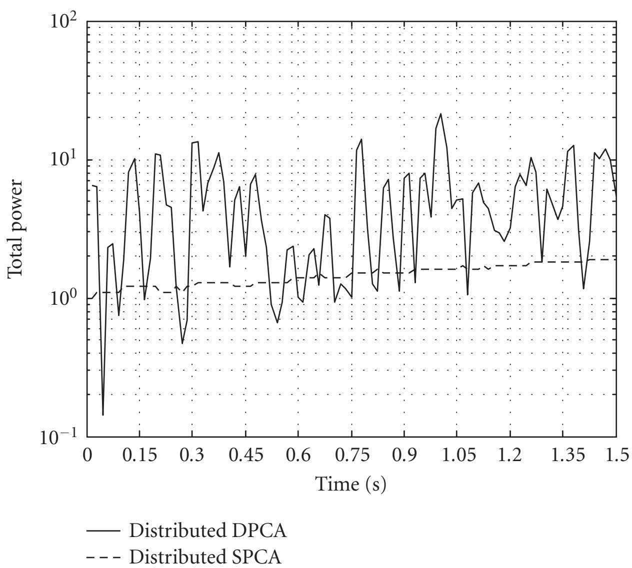

Figure 7: Sum of transmitted power of all mobiles for the distributed DPCA and the distributed SPCA under TV LTF channels.

Example 4. In this example, the performance of the distributed DPCA (30) is compared with the distributed SPCA (34) under stochastic TV LTF channels. With the same parameters as Example 3, in addition to the target SIRs, $\varepsilon_i = 5$ for all users and $a_k = 0.1$. The total transmitted powers of all mobiles using the distributed DPCA in (30) and the SPCA in (34) under stochastic TV LTF channels are shown in Figure 7. Note that the power axis is *logarithmic*. Clearly, the distributed SPCA using stochastic approximations provides better power stability and consumption than the distributed DPCA described in [15–17].

5. CONCLUSION

In this paper, a TV LTF wireless channel model, which captures both the space and time-variations of TV LTF wireless channels, is developed. The dynamics of the TV LTF channels are described by an SDE, which essentially captures the spatiotemporal variations of wireless communication links. The proposed model is more realistic than the standard static models encountered in the literature. The SDE model proposed allows viewing the wireless channel as a dynamical system, which shows how the channel evolves in time and space. In addition, it allows well-developed tools of estimation and identification to be applied to this class of problems [9–11]. An optimal DPCA based on the developed model is proposed. The optimal DPCA is shown to reduce to a simple linear programming problem if predictable power control strategies (PPCS) are used. Iterative distributed DPCAs and SPCAs are used to solve for the optimization problem. The proposed distributed SPCA requires less information than the distributed SPCAs encountered in the literature. Generalizations to PC problems based on convex optimization techniques are provided if PPCS are not assumed, together with outage constraints. These optimizations are the subject of on-going research. Numerical results show that there are potentially large gains to be achieved by using TV stochastic models, and the distributed SPCA provides better power stability and consumption than the distributed DPCA. It should be noted that channel models based on SDEs for STF (Rayleigh and Ricean environments) have been considered in [2, 5], by approximating the Doppler power spectral density of wireless channels.

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